ref. http://aa.quae.nl/en/reken/zonpositie.html
http://www.stargazing.net/kepler/sunrise.html
https://arxiv.org/pdf/1208.1043.pdf

Time

fixed day & mongth for clock — adjustable year

$d := 31$ $\quad$ $month := 5$ $\quad$ $year := 2016$ $\quad$ $\varphi := 52deg = 0.908 \cdot rad$ $\quad$ Latitude

$$J_{cur} := \left[d - 32075 + 1461 \cdot \frac{\left[year + 4800 + \frac{(month - 14)}{12} \right]}{4} \ldots + 367 \cdot \frac{\left[month - 2 - \frac{(month - 14)}{12} \cdot 12 \right]}{12} - 3 \cdot \frac{\left[\frac{\left[year + 4900 + \frac{(month - 14)}{12} \right]}{100} \right]}{4} \right] = 2.457541 \times 10^{6}$$

$J_{cur} := floor(J_{cur}) \cdot day = 2457541 \cdot day$ $\quad$ Current julian day from day, month, year

$J_{2000} := 2451545 \cdot day$ $\quad$ Julian day for start of year 2000

1. The Mean Anomaly

$M_0 := 357.5291 \cdot deg = 6.24005997 \cdot rad$ $\quad$ $M_1 := 0.98560028 \cdot \frac{deg}{day} = 0.01720197 \cdot \frac{rad}{day}$ $\quad$ Mean anomaly coefficients for earth

3 $\quad$ $M := mod\left[M_0 + M_1 \cdot (J_{cur} - J_{2000}), 2\pi \right] = 2.5689$ $\quad$ Mean anomaly

$J_{cur} - J_{2000} = 5.996 \times 10^{3} \cdot day$

2. The Equation of Center

$C_1 := 1.9148 \cdot deg = 0.03341956 \cdot rad$

$C_2 := 0.0200 \cdot deg = 0.00034907 \cdot rad$ $\quad$ equation of centre coefficients for earth

$C_3 := 0.0003 \cdot deg = 0.00000524 \cdot rad$

5 $\quad$ $C := C_1 \cdot \sin(M) + C_2 \cdot \sin(2 \cdot M) + C_3 \cdot \sin(3 \cdot M) = 0.0178$ $\quad$ mean anomaly - Equation of Centre (C)

4. The Perihelion and the Obliquity of the ecliptic

$\Pi := 102.9372deg = 1.79659306 \cdot rad$ $\quad$ ecliptic longitude

$\varepsilon := 23.45deg = 0.40927971 \cdot rad$ $\quad$ Obliquity

5. The Ecliptical Coordinates

8 $L_{sun} := \mathrm{mod}(M + \Pi + \pi, 2\cdot\pi) = 1.2239$ mean longitude

9 $\lambda_{sun} := \mathrm{mod}(L_{sun} + C, 2\cdot\pi) = 1.2417$ ecliptical longitude

6. The Equatorial coordinates

14 $\delta_{sun} := \mathrm{asin}(\sin(\lambda_{sun})\cdot\sin(\varepsilon)) = 0.3861$ declination (north / south)

7. The Observer

24 $H := \frac{2\cdot\pi}{24}\cdot(loc\_time - 12) = 0$ hour angle, time from meridian

$h := \mathrm{asin}(\sin(\varphi)\cdot\sin(\delta_{sun}) + \cos(\varphi)\cdot\cos(\delta_{sun})\cdot\cos(H)) = 1.0493$ alitiude to sun, from horizon (0deg) to zenith (+90deg)

25 $\phi_{sun} := \mathrm{asin}(\mathrm{mod}(-\sin(H)\cdot\cos(\delta_{sun}), \sin(h))) = \blacksquare\cdot deg$ $h = 60.123\cdot deg$ azimuth

Now we have the altitude from the time, we can calculate a table of angle versus time, which can then be referenced in caculating the reflected/refracted and direct light coming from the sun

Atmospheric refraction calculated by Saemundsson's formula

$$R := \frac{0.017deg}{\tan\left(h + \frac{10.3deg}{h + 5.11deg}\cdot 1deg\right)} = \blacksquare\cdot deg$$

WS2812S specs

red = 650nm, 600mcd
green = 520nm, 1200mcd
Blue = 460nm, 300mcd

Astronomical twilight	altitude -18 degrees
Nautical twilight	altitude -12 degrees
Civil twilight	altitude -6 degrees

calculate the spectrum; using SUNCOLOR formuli, then need to evaluate the RGB value by calculating (integrate?) the CIE XYZ values over the calculated spectrum. The XYZ values can then be converted into RGB values for the LEDs

Calculations need to be efficient for 8bit CPU so need to select the bin resolution appropriatly.

ref. https://www.cs.utah.edu/~shirley/papers/sunsky/sunsky.pdf
http://pastebin.com/zVFJ8uZn
https://nicoschertler.wordpress.com/2013/04/03/simulating-a-days-sky/

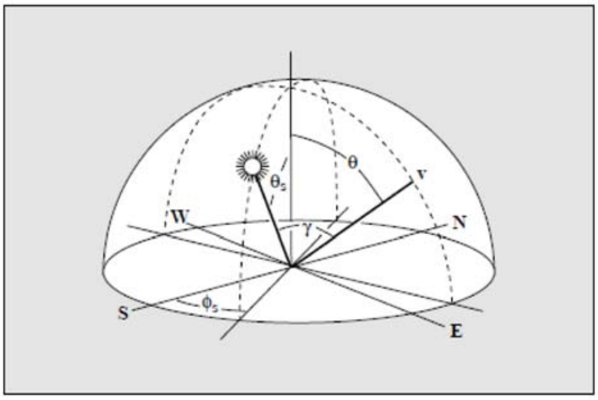

Figure 4: *The coordinates for specifying the sun position and the direction* **v** *on the sky dome.*

gamma function is effective angle from sun, irrespective of location on the circumference.

so simplify to remove azimuth from calculation

α_{sun} = right ascention
δ_{sun} = declination
h = altitude (θ_s)= 90deg - h
ϕ_{sun} = azimuth

zenith at pixel

azimuth at pixel

$\phi_{pix} := 0deg$

turbidity $turb := 1.8$

sun zenith $\theta_{sun} := \frac{\pi}{2} - h = 0.521$

Yxy
012

$$A_{coef} := \begin{pmatrix} 0.1787 \cdot turb - 1.4630 \\ -0.0193 \cdot turb - 0.2592 \\ -0.0167 \cdot turb - 0.2608 \end{pmatrix} \quad B_{coef} := \begin{pmatrix} -0.3554 \cdot turb + 0.4275 \\ -0.0665 \cdot turb + 0.0008 \\ -0.0950 \cdot turb + 0.0092 \end{pmatrix} \quad C_{coef} := \begin{pmatrix} -0.0227 \cdot turb + 5.3251 \\ -0.0004 \cdot turb + 0.2125 \\ -0.0079 \cdot turb + 0.2102 \end{pmatrix}$$

$$D_{coef} := \begin{pmatrix} 0.1206 \cdot turb - 2.5771 \\ -0.0641 \cdot turb - 0.8989 \\ -0.0441 \cdot turb - 1.6537 \end{pmatrix} \quad E_{coef} := \begin{pmatrix} -0.0670 \cdot turb + 0.3703 \\ -0.0033 \cdot turb + 0.0452 \\ -0.0109 \cdot turb + 0.0529 \end{pmatrix}$$

angle between sun and pixel

$$\gamma := acos\left(\sin\left(\theta_{sun}\right) \cdot \sin\left(\theta_{pix}\right) \cdot \cos\left(\phi_{pix} - \phi_{sun}\right) + \cos\left(\theta_{sun}\right) \cdot \cos\left(\theta_{pix}\right)\right) = \blacksquare \cdot deg$$

$$\gamma := acos\left(\sin\left(\theta_{sun}\right) \cdot \sin\left(\theta_{pix}\right) + \cos\left(\theta_{sun}\right) \cdot \cos\left(\theta_{pix}\right)\right) = 0.962$$

luminosity

$$PerezLum(\theta, \gamma, n) := \left(1 + A_{coef_n} \cdot e^{\frac{B_{coef_n}}{\cos(\theta)}}\right) \cdot \left(1 + C_{coef_n} \cdot e^{D_{coef_n} \cdot \gamma} + E_{coef_n} \cdot \cos(\gamma)^2\right)$$

colour

$$Yz := \frac{(4.0453 \cdot turb - 4.9710) \cdot \tan\left[\left(\frac{4}{9} - \frac{turb}{120}\right) \cdot (\pi - 2 \cdot \theta_{sun})\right] - 0.2155 \cdot turb + 2.4192}{(4.0453 \cdot turb - 4.9710) \cdot \tan\left[\left(\frac{4}{9} - \frac{turb}{120}\right) \cdot (\pi)\right] - 0.2155 \cdot turb + 2.4192} = 0.403$$

$$xz := \left(0.00166 \cdot \theta_{sun}^{3} - 0.00375 \cdot \theta_{sun}^{2} + 0.00209 \cdot \theta_{sun}\right) \cdot turb^{2} \ldots + \left(-0.02903 \cdot \theta_{sun}^{3} + 0.06377 \cdot \theta_{sun}^{2} - 0.03202 \cdot \theta_{sun} + 0.00394\right) \cdot turb \ldots + \left(0.11693 \cdot \theta_{sun}^{3} - 0.21196 \cdot \theta_{sun}^{2} + 0.06052 \cdot \theta_{sun} + 0.25886\right) \cdot 1 = 0.251$$

$$yz := \left(0.00275 \cdot \theta_{sun}^{3} - 0.00610 \cdot \theta_{sun}^{2} + 0.00317 \cdot \theta_{sun}\right) \cdot turb^{2} \ldots + \left(-0.04214 \cdot \theta_{sun}^{3} + 0.08970 \cdot \theta_{sun}^{2} - 0.04153 \cdot \theta_{sun} + 0.00516\right) \cdot turb \ldots + \left(0.15346 \cdot \theta_{sun}^{3} - 0.26756 \cdot \theta_{sun}^{2} + 0.06670 \cdot \theta_{sun} + 0.26688\right) \cdot 1 = 0.255$$

perceived luminosity of the light

$$Yxy_Y := Yz \cdot \frac{PerezLum(\theta_{pix}, \gamma, 0)}{PerezLum(0, \theta_{sun}, 0)} = 2.81$$

$$Yxy_x := xz \cdot \frac{PerezLum(\theta_{pix}, \gamma, 1)}{PerezLum(0, \theta_{sun}, 1)} = 0.298$$

$$Yxy_y := yz \cdot \frac{PerezLum(\theta_{pix}, \gamma, 2)}{PerezLum(0, \theta_{sun}, 2)} = 0.307$$

Convert from CIE Yxy to CIE XYZ

$$X := \frac{Yxy_x}{Yxy_y} \cdot Yxy_Y = 2.721 \qquad Y := Yxy_Y = 2.81 \qquad Z := \frac{1 - Yxy_x - Yxy_y}{Yxy_y \cdot Yxy_Y} = 0.457$$

CIE XYZ to RGB conversion (`CIEsystem colour system`) https://www.fourmilab.ch/documents/specrend/

$xr := 0.7355$ $yr := 0.2645$ $zr := 1 - (xr + yr) = 0$

$xg := 0.2658$ $yg := 0.7243$ $zg := 1 - (xg + yg) = 9.9 \times 10^{-3}$

$xb := 0.1669$ $yb := 0.0085$ $zb := 1 - (xb + yb) = 0.825$

$xw := 0.33333333$ $yw := 0.33333333$ $zw := 1 - (xw + yw) = 0.333$

RGB matrix before scaling white

$rx := (yg \cdot zb) - (yb \cdot zg) = 0.597$ $ry := (xb \cdot zg) - (xg \cdot zb) = -0.218$ $rz := (xg \cdot yb) - (xb \cdot yg) = -0.119$

$gx := (yb \cdot zr) - (yr \cdot zb) = -0.218$

$gy := (xr \cdot zb) - (xb \cdot zr) = 0.606$

$gz := (xb \cdot yr) - (xr \cdot yb) = 0.038$

$bx := (yr \cdot zg) - (yg \cdot zr) = 2.619 \times 10^{-3}$

$by := (xg \cdot zr) - (xr \cdot zg) = -7.281 \times 10^{-3}$

$bz := (xr \cdot yg) - (xg \cdot yr) = 0.462$

white scaling factors

$$rw := \frac{(rx \cdot xw) + (ry \cdot yw) + (rz \cdot zw)}{yw} = 0.261$$

$$gw := \frac{(gx \cdot xw) + (gy \cdot yw) + (gz \cdot zw)}{yw} = 0.426$$

$$bw := \frac{(bx \cdot xw) + (by \cdot yw) + (bz \cdot zw)}{yw} = 0.458$$

XYZ to RGB scaled to white

$rx := \frac{rx}{rw} = 2.28783849$

$ry := \frac{ry}{rw} = -0.83336768$

$rz := \frac{rz}{rw} = -0.4544708$

$gx := \frac{gx}{gw} = -0.51165138$

$gy := \frac{gy}{gw} = 1.42275838$

$gz := \frac{gz}{gw} = 0.088893$

$bx := \frac{bx}{bw} = 0.00572041$

$by := \frac{by}{bw} = -0.01590685$

$bz := \frac{bz}{bw} = 1.01018641$

RGB of desired point

$RGB_0 := (rx \cdot X) + (ry \cdot Y) + (rz \cdot Z) = 3.676$

$RGB_1 := (gx \cdot X) + (gy \cdot Y) + (gz \cdot Z) = 2.647$

$RGB_2 := (bx \cdot X) + (by \cdot Y) + (bz \cdot Z) = 0.433$

$r := 3.2406 \cdot X - 1.5372 \cdot Y - 0.4986 \cdot Z = 4.27$

$g := -0.9689 \cdot X + 1.8758 \cdot Y + 0.0415 \cdot Z = 2.654$

$b := 0.0557 \cdot X - 0.2040 \cdot Y + 1.0570 \cdot Z = 0.062$

constrain RGB to colour gamut

```
RGB := | if  RGB_0 ≤ 0 ∨ RGB_1 ≤ 0 ∨ RGB_2 ≤ 0
       |    | w ← −min(0, RGB_0, RGB_1, RGB_2)
       |    | RGB_0 ← RGB_0 + w
       |    | RGB_1 ← RGB_1 + w
       |    | RGB_2 ← RGB_2 + w
       | return  RGB
```

$$RGB = \begin{pmatrix} 3.676 \\ 2.647 \\ 0.433 \end{pmatrix}$$

normalise RGB

$$RGB := \begin{array}{|l} a \leftarrow \max(1, RGB_0, RGB_1, RGB_2) \\ \text{for } i \in 0..2 \\ \quad RGB_i \leftarrow \dfrac{RGB_i}{a} \\ \text{return } RGB \end{array}$$

$$int(x) := round(x, 0)$$

1,000 lux overcast midday sunlight
40 lux overcast sunrise

Sky RGB colour

$$RGB = \begin{pmatrix} 1 \\ 0.72 \\ 0.118 \end{pmatrix}$$

$$R_{ed} := \begin{array}{|l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow 255 \cdot RGB_0 \\ \text{return } out \end{array} \qquad G_{reen} := \begin{array}{|l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow 255 \cdot RGB_1 \\ \text{return } out \end{array} \qquad B_{lue} := \begin{array}{|l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow 255 \cdot RGB_2 \\ \text{return } out \end{array}$$

$R_{ed}, G_{reen}, B_{lue}$

$loc\_time \equiv 12$ 24 hour clock

$\theta_{pix} \equiv 85deg$

$theta\_max := 30deg = 0.524$

$\theta_{sun} = 0.521$

$factor := \cos\left[\left(\theta_{sun} - theta\_max\right)\cdot 1.3\right]\cdot 8 = 8$

brightness from -6 deg to max sun sin function

scale colour to intensity

$RGB := (512\cdot RGB) = \begin{pmatrix} 512 \\ 368.664 \\ 60.317 \end{pmatrix}$

split 512bit number into odd and even bits to give two 256 bit numbers that can be sent to pair of LEDs

$RGB = \begin{pmatrix} 512 \\ 368.664 \\ 60.317 \end{pmatrix}$

$diff := 64$

ref. https://hackaday.com/2016/08/23/rgb-leds-how-to-master-gamma-and-hue-for-perfect-brightness/

gamma correct RGB

$gamma := 1.8$

$512^{\frac{1}{gamma}}\cdot factor = 255.999$

$$RGB := \left| \begin{array}{l} \text{for } i \in 0..2 \\ \quad RGB_i \leftarrow \text{int}\left[\left(RGB_i\right)^{\frac{1}{gamma}}\cdot factor - diff\right] \\ \text{return } RGB \end{array} \right.$$

$RGB = \begin{pmatrix} 192 \\ 149 \\ 14 \end{pmatrix}$

$$R_{ed} := \left| \begin{array}{l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow RGB_0 \\ \text{return } out \end{array} \right.$$

$$G_{reen} := \left| \begin{array}{l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow RGB_1 \\ \text{return } out \end{array} \right.$$

$$B_{lue} := \left| \begin{array}{l} \text{for } i \in 0..50 \\ \quad \text{for } j \in 0..50 \\ \quad\quad out_{i,j} \leftarrow RGB_2 \\ \text{return } out \end{array} \right.$$

$R_{ed}, G_{reen}, B_{lue}$